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A new multipath channel estimation and mitigation using annihilation filter combined tracking loop implementation in software GPS receivers

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Abstract

The multipath effect causes severe degradation in the positioning of commercial GPS receivers. Due to multipath error, the positioning accuracy could reach a few of 10 m. If the cumulative Multipath delay is less than 0.1–0.35 chips, then it is difficult to mitigate in GPS receivers. This causes severe degradation in GPS signals and can cause a measurement bias. To alleviate this problem, the estimation of multipath parameters using annihilating filter and its mitigation in the GPS tracking loop is proposed in this work. The estimation of randomly generated multipath signals can be performed in the receiver with a lower sampling rate when compared to the larger bandwidth of the GPS baseband signal. Here, the frequency components of the multipath signal in superimposed complex exponentials have been transformed from the time delay and the amplitude of the path observables. The Rayleigh fading model in the urban scenario has been simulated in which the amplitude and the phase of the number of paths (i.e. the frequency component of superimposed complex exponentials) are set and this fading signal is convolved with GPS signal that forms the multipath faded signal. In the GPS receiver post-processing stage, with the help of the annihilation filter, the multipath components are estimated, then an inverse/adaptive filter and compensation technique are further applied to mitigate the multipath component. The mean square error with the different number of paths with noisy environments is analyzed utilizing the cadzow denoising algorithm. The simulation results of the proposed technique employed in the tracking module of the software GPS receiver under severe multipath conditions indicate a substantial enhancement in the performance of the GPS receiver with minimal code and carrier phase error when compared to the least squares and adaptive blind equalization channel techniques. Moreover, the positioning accuracy is also calculated with the inclusion of multipath components in two satellites out of six satellites used in the simulation, the results showed that the annihilation filter improved the mean position accuracy up to 9.3023 m.

Keywords: multipath, GPS, annihilation filter, FRI (finite rate of innovation), cadzow filter

(Some figures may appear in colour only in the online journal)

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1. Introduction

Indeed, the technical advancements in global navigation satellite systems (GNSS) receiver for providing precise positioning is the mandate one to improve the positioning error in civilian applications. Global positioning system (GPS) is the widely used navigation system across the globe and based on the error budget of commercial grade receivers, the multipath error and receiver noise are classified under the category of the non-correctable error. In this paper our subject of interest is diminishing the multipath positioning error, since it easily cannot be eliminated with the help of relative positioning or short baseline between two receivers [1].

Multipath is the predominant factor influencing the position accuracy of the GPS satellite reception and sometimes this leads to the re-acquisition of GPS signal and causing the positioning error up to several meters. When the transmitted signal reached at the antenna of the GPS receiver through multiple paths in the form of reflection, scattering, refraction, and diffraction from surrounding obstacles, such as buildings, ground, water surfaces, vehicles, mountains, trees, etc affect the direct path which leads to the fading effect fluctuating the signal power. Multipath signals always arrive at the receiver in a delayed manner compared to line-of-sight (LOS) path owing to the lengthier travel paths caused by the above-mentioned reasons. In navigation receivers, the signal of interest is only the first path known as LOS path. However, knowledge about the other multipath can help in reducing multipath interference. This is different from communication receivers, in order to increase the receiver performance where energy from several paths (LOS and NLOS) can be combined [2]. Many multipath algorithms have been proposed to reduce the effects of multipath, to mention few works at different modules of the receiver, especially at the RF front-end level, the methods like antenna polarization, antenna pattern shaping/adaptive antenna arrays, choke ring antenna, etc are used to minimize the multipath effect before the signal entering the receiver. Some of the works mentioned in the literature [3–7] significantly nullifying the effect of multipath on the antenna stage itself. Similarly, in the post-processing i.e. baseband stage, particularly in the signal processing chain of receivers (acquisition/tracking module), the multipath effect has been diminished. In code tracking stage, the mitigation methods such as correlation-based maximum likelihood (ML), non-linear transforms, deconvolution, super-resolution/eigenvalue decomposition techniques, etc have also been implemented in GPS receivers [8–22] that works well to combat the multipath effect. In the navigation module of the receiver, few works associated with obstacle mapping [23], optimal combinations of function for carrier to noise ratio (C/N_0) measurements [24], and the correction in pseudorange observable has been performed using wavelet transform technique implemented in the navigation processing and positioning module for kinematic applications [1]. The elimination of NLOS can also be done by utilizing the fisheye camera in choosing the satellites in view based on the C/N_0 from sky view in the urban dense environment [25].

GPS signal reception, if the early-late correlator spacing in the code tracking loop is more than one chip then this may introduce the positioning error up to 100 m [26]. Suppose the signals which are reflected off surfaces very close to the GPS receiver, say a wall next to the receiver or the ground below, that leads to cumulative multipath delay less than 0.35 chips, then it is classified as short delay multipath. These are very hard to mitigate. Therefore, minimum correlator spacing in the conventional GPS receiver should be kept as 0.10 chips in code delay of three chips typically ranging from -1.5 chips to $+1.5$ chips [27, 28]. To handle the short delay multipath problem, many literatures have dealt with blind multipath channel equalization employing least squares [29], adaptive filters [30], *a posteriori* estimation of the tracking errors [31], rake-based code and carrier tracking loop [32] for estimating the multipath delays, phases, and amplitudes using the traditional approach but if the paths that are very close within 0.1–0.35 chips apart and also the number of multipath components are more than ten, then the estimation of amplitude and the phase becomes inaccurate and consequently these observables used in the adaptive filter/equalizer for predicting the channel impulse response is imprecise to filter the correlation peak and difficult to mitigate the multipath component. To address these problems, we have implemented an annihilation filter-based channel estimation and mitigation in the software GPS receiver tracking loop design.

The main contribution of the work is summarized below.

(a) The sampling of received multipath signal is performed with lesser number of samples than a conventional Nyquist sampling based on the rate of innovation (ROI).
(b) The annihilating filter is implemented in GPS tracking loop to precisely estimate the amplitude and the phase of the paths and further applied to inverse/adaptive filter to mitigate the multipath component.
(c) The mean square error estimation of different number of paths using the proposed technique is compared with the least square technique.
(d) Tracking loop parameters and the positioning accuracy are computed with and without multipath components to justify the superiority of the proposed method.

The organization of the paper is structured as follows. Next section illustrates an explanation about varieties of related works relevant to multipath mitigation. Section 2 explains least square approach followed by equalizer/inverse filter design in multipath channel estimation. A description about proposed technique and its mathematical background has been mentioned in sections 3 and 4 respectively. The results of the proposed technique in GPS tracking stage are presented in section 5. Eventually, a conclusion section has been provided in section 6.

1.1. Related works

The correlation-based technique used in multipath mitigation relies on the number of correlators used in the receiver which comprises of number of the correlators 3, 4, 5, and greater than 7, respectively. The wide/narrow early minus late (wEML/nEML) are broadly used methods employing three correlators. The slope based method utilizing four correlators listed in the literature are (E1/E2) early 1/early 2 trackers, *a posteriori* multipath estimator, slope based multipath estimator (SBME) that are effectively nullifying the effect of multipath [33, 34]. On the other hand, the techniques of ($\Delta\Delta$) double delta, early late slope, multipath estimation technique (MET) have also been used with five correlators [35–37]. The double delta-based method further classified into high resolution correlator (HRC), very early very late [36], strobe correlator [38, 39], and pulse aperture correlator [40] for compensating the code phase errors. More than seven correlators used in GPS receivers for C/A-code and the carrier phase observations are multiple gate delay, reduced search maximum likelihood, and multipath estimating delay locked loop [41–43]. The estimation of multipath code delay in the urban scenario is carried out by Kumar and Singh [44] used a double differentiated correlation function-based histogram approach. The beamforming technique using bidirectional least mean squared algorithm is also used in GPS receivers to minimize the multipath effects implemented by Siridhara and Ratnam [45].

Other techniques involved in code tracking are nonlinear methods such as TK (Teager Kaiser), quadratic optimization. Some of the deconvolution methods mentioned in the literature are least squares (LS), projection onto convex sets (POCS) utilizes the unconstrained optimization algorithm to retrieve the LOS path from the composite signal [46–52]. The combined hybrid methods using both nonlinear and deconvolution generalized TK also reduce the code phase tracking error to a certain extent. Super-resolution/eigenvalue decomposition approaches (e.g. ESPRIT, MUSIC, EM) works at chip-level using the matrices and looking for maxima formed by different time-shifted values of the code and the eigenvalue decomposition [33]. The complexity reduction has been accomplished through various approaches corresponding to Newton Gauss Vision Correlator patented by Novatel manufacturer [53]. Some of the other statistical methods like the Bayesian technique also work well on the positioning stage of the GPS receiver [54].

Multipath mitigation uses finite impulse response (FIR) adaptive filter, blind channel equalizer and least squares method utilize a FIR LMS, RLS adaptive filter/channel equalizer to mitigate the effect of multipath in the baseband stage [28–30]. It is shown that the estimation of the multipath delays can be obtained by adjusting the tap-weight vector of the adaptive filter/equalizer. However, the estimate of multipath observable lack in accuracy of amplitude and phase by closely spaced multipath components. This problem needs to be addressed with a precise estimation of path parameters when more than ten multipath components and the spacing between the paths is also very close that is lesser than 0.35 chips.

The limitation of usage of several multipath mitigation methods is listed below. In RF front-end level, the methods like choke rings-based technique, antenna polarization work well in static/unchanging scenarios, but difficult to be used in dynamic scenarios (fast-changing multipath). A correlator-based technique used in GPS receiver has low complexity, but it works moderately at low C/N_0 and good at high C/N_0. LS, POCS relies on parameter optimization according to the environment. But there is a trade-off between complexity and performance. Some of the methods have high complexity and work best at high C/N_0 but the major limitation is sensitive to noise. Unified studies of all possible multipath mitigation algorithms are still rare. Classical/simplest approaches (EML and HRC) are typically preferred by manufacturers. Always tradeoffs complexity-performance. Some approaches like SBME deal very good with very short delays but have lower performance in moderate/long delays. ML-based have good performance but higher complexity; among the lowest complexity ones, two-stage estimators and SBME are most promising.

2. Least square technique of channel estimation

The blind channel estimation of multipath in GPS receivers is dealt with least squares and/or adaptive filter type of equalization. According to the least-squares technique, the DLL discriminator output (d) of the multipath component is the convolution of the ideal discriminator function (g) and the channel's impulse response (h) is given by:

$$g * h = d.$$

The multipath discriminator output (d) is sampled in the range between (N_1, N_2) and the quantizer upper limit of the discriminator is fixed as M, from this, the matrix represents the above equation can be represented as:

$$\underbrace{\begin{bmatrix} g_{N1} & g_{N1-1} \cdots & g_{N1-M} \\ g_{N1+1} & g_{N1} \cdots & g_{N1-M+1} \\ \vdots & \vdots \ddots & \vdots \\ \vdots & \vdots & \\ g_{N2} & g_{N2-1} \cdots & g_{N2-M} \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{M-1} \\ h_M \end{bmatrix}}_{H} = \underbrace{\begin{bmatrix} d_{N1} \\ d_{N1+1} \\ \vdots \\ d_{N2-1} \\ d_{N2} \end{bmatrix}}_{D}.$$

The ideal discriminator function can be stored as a lookup table in a GPS receiver. With the help of the least squares technique, multipath components can be easily estimated. The multipath component is given by:

$$\tilde{H} = \left(G^H G\right)^{-1} G^H D.$$

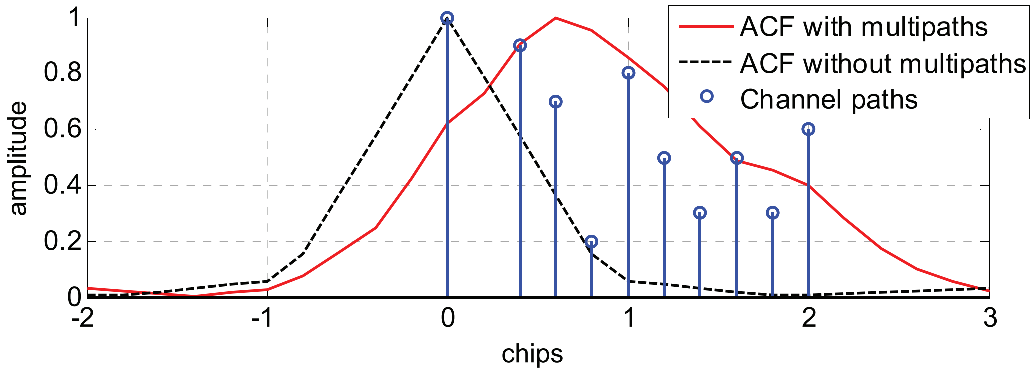

Figure 1. Correlation peak—multipath influenced scenario under ten multipath components.

2.1. Inverse filter/equalizer design

Once the multipath channel impulse response is estimated then the impulse response of the optimized equalization filter needs to be obtained to mitigate the multipath. One can design the equalization filter impulse response as $G = [g_0, g_1, g_2, \ldots\ldots g_p,]^{\mathrm{T}}$ then the delta function is given by $\delta = [0,0,0,\ldots 1,.,0,0,0]^{\mathrm{T}}$. This delta function is a delayed signal positioned anywhere in the time scale and this should be the delay of the ideal impulse response because the multipath channel response has a delay. The equalization filter must be an advanced system that is a non-causal one to compensate for the effect of the multipath

$$Q = \begin{bmatrix} \tilde{h}_0 & 0 & \ldots 0 \\ \tilde{h}_1 & \tilde{h}_0 & \ldots 0 \\ \tilde{h}_2 & \tilde{h}_1 & \ldots \tilde{h}_1 \\ \vdots & \vdots & \vdots \\ \tilde{h}_M & \tilde{h}_M & \ldots \tilde{h}_M \end{bmatrix}.$$

The output of the equalization filter is given by:

$$G = \left(Q^H Q\right)^{-1} Q^H \delta.$$

Now the DLL discriminator output is given to the equalization filter, the performance is almost similar to an ideal discriminator. But this technique does not work well if the separation between the paths is more than 0.35 chips and also a limited number of paths. As shown in figure 1, while considering the ten numbers of paths having closely spaced intervals about 0.1–0.35 chips apart, the mean square error between the actual and estimated channel parameters (amplitude and phase) is beyond 0.2 as depicted in figure 2. It is not capable enough to get the correct estimation of path observables. This leads to a loss of lock in the tracking stage of the signal in the GPS receiver. In this scenario, the method of accurately estimating the channel observable under severe multipath conditions is required.

3. Proposed technique

The finite rate innovation technique (FRI) concept introduced by Vetterli has been widely used in many applications such as spike detection, channel and, timing estimation [55–58], etc. The theorem states that the periodic signal $x(t)$ with 'K' Diracs on the interval having $2K$ degrees of freedom then taking '$2K + 1$' samples from a lowpass version of bandwidth allows to perfectly recover $x(t)$. In this work, the baseband GPS signal is transmitted on the channel while the sampling is done at a very low sampling rate of the received multipath faded signal that is determined based on the ROI. This concept of ROI applies to the multipath scenario, i.e. in order to retain the original signal, the minimum number of samples should be kept at least more than twice the time of the number of paths that are to be identified. In this work, Initially, the composite multipath signal is estimated through an annihilation filter combined reduced rank cadzow filter, from this, the amplitude and the phase of the multipath signal have been estimated. Once the estimated observables are obtained, which could be given as an initialization weight parameter (amplitude and phase composite as $a_n(t)\,\mathrm{e}^{-j\theta_n(t)}$) of the filter coefficient of the inverse filter/equalizer. Then the filter weights are updated based on the RLS algorithm. The LOS path is detected in the final stage of the mitigation by performing the subtraction operation of the strongest component of the composite signal. In general, at the tracking stage of the GPS receiver, after the stripping process of the carrier signal, the in-phase and quadrature-phase signals are multiplied with the early, late and prompt replicas of the locally generated PRN code then the outputs are filtered through the accumulation process. As a result of this, the delay lock loop discriminator's output function in the presence of multipath is the convolution of the ideal discriminator function with the channel's filter coefficient. With the help of the least square technique, the channel impulse response is estimated. But, in this work, the composite multipath signal in the form of phase and the amplitude is estimated with the help of an annihilation filter. The overall view of the proposed method is depicted in figure 3.

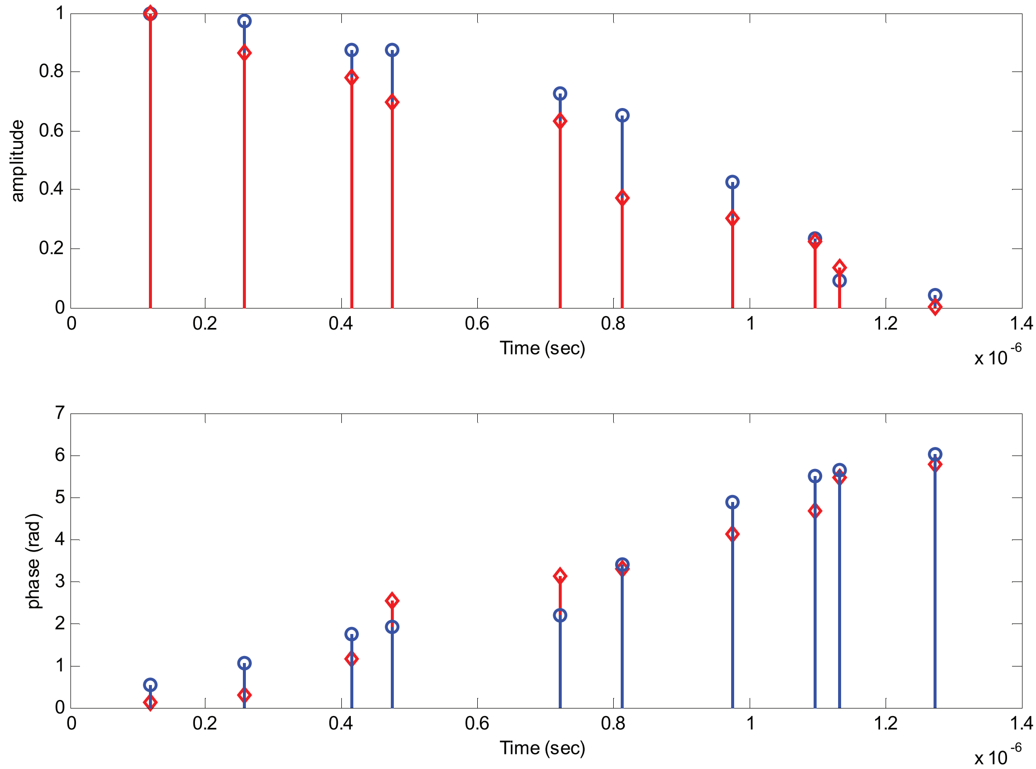

Figure 2. Amplitude and phase observables using least square method.

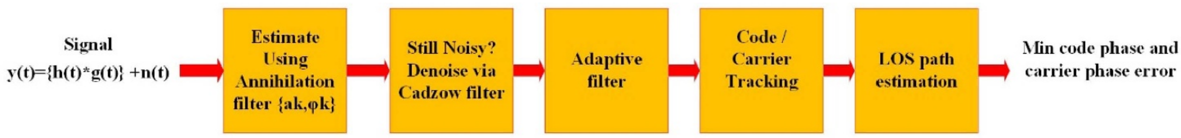

Figure 3. Overall view of the proposed block diagram.

4. Problem formulation and algorithm for retrieving the amplitudes and time delay and from the composite signal

The transmitted GPS signal is given as:

$$x(t) = g(t) \times v_1(t) = \sum_{i \in Z} g_i(t)\, v_1\left(t - i\frac{\tau}{L}\right) \tag{1}$$

where $g(t) = \sum_{l=1}^{L} [C_l(t-\tau_l) \oplus D_l(t-\tau_l)] \sin[2\pi(f_{RF} + f_{d,l})t]$.

$g(t)$ denotes transmitted GPS signal which comprises of pseudorandom signal C_l modulated onto the RF carrier signal f_{RF} with a Doppler frequency f_d. Consider the period of the GPS signal is 'τ'. The signal is then applied to the first low-pass filter $v_1(t)$ having the bandwidth of L/τ. The bandwidth of the transmitted signal is considered to be very large i.e. $B_1 = \frac{L}{\tau}$. Then the signal is transmitted to the multipath channel having impulse response $h(t)$ is given by $y(t) = x(t) \times h(t)$.

A channel model for the multipath environment is given by the impulse response $h(t)$ which is a stream of Diracs delta function given by equation (2)

$$h(t) = \sum_{k=1}^{L_p} a_k \delta(t - t_k - kT_c) \tag{2}$$

where
L_p = number of paths,
t_k = path delay,
a_k = normalized amplitude k-th path,
$\delta(t)$ = Dirac pulse,
T_c = chip duration.

The resultant signal $y(t)$ is not bandlimited, but the k pair of search parameters (a_k, t_k) need to be identified to perfectly reconstruct it. The classical sampling theory fails to do it and the complete restoration of this type of signal can be made possible based on FRI theory proposed by Vetterli *et al*. Hence the GPS receiver requires channel impulse response to identify the amplitude and accurate timing estimation of the multipath signal.

By applying the signal to the kernel function (sinc filter or sum of sinusoids or spline etc) that can be able to reproduce the exponentials. Figure 4 shows the FRI model where this type of kernel is usually modelled as filtering (LPF-2) followed by a sampling stage. Then estimation of search

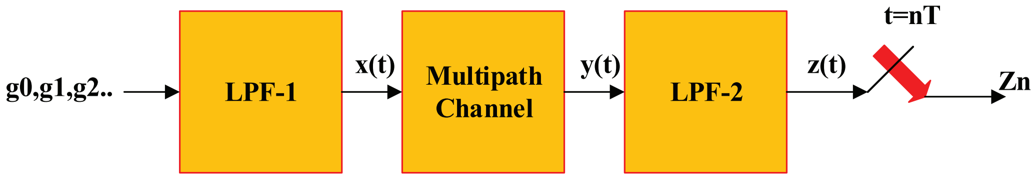

Figure 4. FRI model for channel estimation.

parameters t_k and a_k can be obtained from a small number of samples by keeping the bandwidth of the kernel filter $B_2 = \frac{N}{\tau}$. The equivalent operation of computing the inner product between $y(t)$ and $\phi(t/T - n)$ is attained by filtering signal $y(t)$ with $v_2(t) = \varphi(-t/T)$ and retrieving samples at time instants $t = nT$

$$z(t) = y(t) \times v_2(t) = \int_{-\infty}^{\infty} y(\tau)\, v_2(t-\tau)\, \mathrm{d}\tau \tag{3}$$

and we have:

$$z(t) = \int_{-\infty}^{\infty} y(\tau)\, \varphi\left(-\frac{t-\tau}{T}\right) \mathrm{d}\tau.$$

Now, sampling $z(t)$ at $t = nT$ yields to:

$$\begin{aligned} z_n = z(t)\,|t = nT = & \int_{-\infty}^{\infty} y(\tau)\, \varphi\left(\frac{\tau}{T} - n\right) \mathrm{d}\tau \\ = & \left\langle y(t), \varphi\left(\frac{\tau}{T} - n\right)\right\rangle. \end{aligned} \tag{4}$$

Given the samples z_n, To accomplish the required degrees of freedom k pair of coefficients (ak, tk) by combining these samples with coefficients $c_{m,n}$, then:

$$\sum_n c_{m,n} z_n = \sum_n c_{m,n} \langle y(t), \varphi(t-n)\rangle$$

$$= \left\langle y(t), \sum_n c_{m,n} \varphi(t-n)\right\rangle = \int_{-\infty}^{\infty} y(t)\, e^{\alpha_m t} \mathrm{d}t = s_m. \tag{5}$$

The sum of exponential terms is found by combining the coefficients $(c_{m,n})$ with the multiplication of filtered signal and sampled stream of Diracs. The locations of the stream of Diracs have been retrieved by filtering the second stage with the exponential reproducing function $\phi(t)$ allowed to solve the problem. The multipath signal is in the form of a stream of Diracs and λ is the parameter of the coefficients $\alpha_m = (\alpha_0 + m\lambda)$. After substituting the value of $y(t)$ and with the help of shifting property of delta function the equation (5) can be written as:

$$s_m = \sum_{k=0}^{L_p} a_k \mathrm{e}^{\alpha_m t_k} = s_m = \sum_{k=0}^{L_p} a_k \mathrm{e}^{(\alpha_0 + m\lambda) t_k} = \sum_{k=0}^{L_p} b_k {u_k}^m \tag{6}$$

where $b_k = a_k \mathrm{e}^{\alpha_0 t_k}$ and $u_k = \mathrm{e}^{\lambda t_k}$.

The above signal is in the form of s_m has sum of L_p exponentials, $a_k \in \mathbb{R}, u_k \in \mathbb{C}$. Now, the problem of retrieving b_k and u_k from the sequence s_m. One of the searching parameters b_k is a linear one, but u_k is a nonlinear one. Therefore, we apply the annihilation filter method to solve this problem of difficulty in finding the nonlinear terms.

A filter $A(m)$ is designed that annihilates the exponential signal termed as annihilation filter of the signal $s(m)$ where,

$$A(m) * s(m) = 0 \forall m \in N \tag{7}$$

$$A(z) = \prod_{k=0}^{L_p - 1} \left(1 - u_k z^{-1}\right) = \sum_{k=0}^{L_p} A(k)\, z^{-k}. \tag{8}$$

The roots of the annihilation filter or zeros of the filter are equal to the exponential signal. From the signal s_m, finding the values of a_k and u_k implies the construction of the annihilating filter method.

Construct the annihilating filter:

$$z(n) = A(m) \times s(m) = \sum_{k=0}^{L_p} A(k)\, s(m-k) = 0. \tag{9}$$

If the roots $\{u_k\}_{k=1}^{L_p}$ of the filter $A(m)$ are same as the L_p exponentials of $s(m)$, then $A(m)$ annihilates $s(m)$, i.e. $A(m) \times s(m) = 0$.

By considering 'S' be the Toeplitz matrix, in matrix format, the above linear equations can be expressed as:

$$\begin{bmatrix} \vdots & \vdots & \vdots \\ s(L_p) & s(L_p - 1)\ldots & s(0) \\ s(L_p + 1) & s(L_p)\ldots & s(1) \\ \vdots & \vdots & \vdots \\ s(2L_p - 1) & s(2L_p - 2) & s(L_p - 1) \end{bmatrix} \begin{bmatrix} A(0) \\ A(1) \\ \vdots \\ \vdots \\ A(L_p) \end{bmatrix} = 0. \tag{10}$$

The system of equations has a unique solution when $\mathrm{rank}(S)$ = number of paths (L_p), if and only if, at least $2L_p$ values of $s(m)$ are available.

Find the annihilating filter for $s(m)$ from which find $\{u_k\}$ by polynomial rooting, once $\{u_k\}$ values are found then values of

Algorithm 1. Annihilation filter design.

Input: Multipath composite signal, number of paths (L_p), path mormalized amplitude (a_k), path phase φ (deg), No. of samples ($N = 2L_p + 1$), SNR:

(a) generate the multipath composite signal with AWGN noise $y(t) = \{h(t) * g(t)\} + n(t)$.
(b) Transform the time domain signal into the frequency domain equivalent one.
(c) Construct annihilation filter $A(m) \times S(m) = 0$.
(d) Determine the annihilating filter coefficients by computing the SVD of A and select the eigenvector $[a_0, a_1, \ldots\ldots a_K]^{\mathrm{T}}$ corresponding to the smallest eigenvalue.
(e) Estimate the phase $\phi k = \frac{1}{2\pi} \angle \text{roots}\,(A)$.
(f) Estimate the amplitude a_k by solving the van der Monde system equation.

Output: Estimated values of phase and amplitude of multipath components.

$\{a_k\}$ are obtained by solving linear system equation as given by

$$\underbrace{\begin{bmatrix} 1 & 1\ldots & 1 \\ u_0 & u_1\ldots & u_{k-1} \\ \vdots & \vdots\ldots & \vdots \\ \vdots & \vdots\ldots & \vdots \\ {u_0}^{k-1} & {u_1}^{k-1}\ldots & {u_{k-1}}^{k-1} \end{bmatrix}}_{\text{Amplitude}} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ \vdots \\ a_{k-1} \end{bmatrix}}_{\text{Exponent}} = \begin{bmatrix} s(0) \\ s(1) \\ \vdots \\ \vdots \\ s(k) \end{bmatrix}. \tag{11}$$

The system becomes an overdetermined one if the matrix S is rank deficient one with rank K, therefore by setting initial coefficient $a_0 = 1$, the system admits a unique solution. The solving roots of the polynomial $H(z)$ lead to finding the locations of t_k.

4.1. Reduced rank cadzow de-noising algorithm for retrieving the samples yn from faded signal affected by noise

If the estimated signal parameters are still noisy then equation (9) may not be equal to zero, in such situations, the reduced rank cadzow filter is further used to excise the noise. The reduced rank cadzow de-noising algorithm is illustrated below.

In algorithm 2 of the cadzow denoising filter, the embedded matrix can be formulated into three types namely autocorrelation, co-variance, and forward-backward methods. We have used the autocorrelation method since it gives good de-noising capability compared with other methods in lower SNR conditions. The autocorrelation embedded matrix is in the form of the Toeplitz matrix structure so it is used in the rank restoration step of the algorithm. In the case of designing an adaptive filter, the initial weight of the filter is the estimated multipath signal parameters. As shown in figure 5, the multipath amplitude in complex form is represented by the time delay of the filter taps. The multipath coefficients $[a_1, a_2, a_3, \ldots\ldots. a_N]$ represents amplitude and $[\theta_1, \theta_2, \theta_3, \ldots\ldots. \theta_N]$ represents the multipath phase vector. The initialization of the filter taps weight vector is given as $a_1(t)\, e^{-j\theta_1(\mathrm{t})}, a_2(t)\, e^{-j\theta_1(\mathrm{t})}, a_3(t)\, e^{-j\theta_1(\mathrm{t})}, a_4(t)\, e^{-j\theta_1(\mathrm{t})} \ldots\ldots\ldots \ldots, a_N(t)\, e^{-j_N(\mathrm{t})}$ in complex form.

The code delay of 1.5 chips is fixed with the 0.01 chips minimum correlator spacing resulting in 151 delay taps that are used to set the order of the filter which is based on the time delay resolution of the multipath. The delay between the paths is determined by the relative phase difference $\Delta\phi = \phi_2 - \phi_1$ which is a function of frequency and the differential path length $\Delta d = d_2 - d_1$. It is given by $d_2 - d_1 = \frac{\Delta\phi}{2\pi}\lambda$. Where λ is the GPS signal's wavelength which is considered to be 20 cm. Therefore, 151 delay taps are placed based on the spacing between the adjacent paths. As shown in figure 6, the input of the adaptive filter is considered to be the correlation output of the multipath signal and the ideal free multipath signal is used as the desired output signal. The tap weights are updated with the help of the RLS algorithm when the adaptive filter approaches channel impulse response then the error signal converges zero.

The overall implementation is shown in figure 6. In this, a modified SOS-based fading simulator with randomly generated ten number of paths is convolved with the GPS signal with different levels of AWGN noise which is fed to the tracking channel of the GPS receiver and the annihilation filter. The output of the adaptive filter is applied to the compensator to obtain a correlation function for each multipath component. Further, it is subtracted from the received signal's correlation function which brings the decomposition of the dominant path (LOS) from multipath components. This filtered discriminator output allows the tracking loops to lock the GPS signal with the correct code phase and carrier frequency.

4.2. Computation complexity analysis

The algorithm complexity of the annihilation filter has been computed in this section. The initial step of determining the $2K + 1$ spectral value of $x(t)$ requires $O(K\log K)$ computations and the second step of computing the Toeplitz matrix having 'K' annihilating filter coefficients 'hk' takes $O(K^2)$ operations. Similarly, the next step of finding the roots of the z—transform which lead to the K locations t_k would require $O(K^3)$ operations, and the final step of solving the van der Monde system of equations which leads to the K weights 'xk' also require $O(K^3)$ operations.

5. Results and discussion

In this simulation, in order to get the urban scenario, the GPS L1 signal with a nominal signal strength of 45 dB-Hz

Algorithm 2. Reduced rank cadzow denoising algorithm for noise excision.

Input: Noisy GPS observables, $y_n = y_0, y_1, \ldots\ldots y_{N-1}$ with size of [1 × N], M, rank, No. of iterations (k).

Step [1]: Selection of embedded matrix:
(a) Choose M value, where typically $N > M$.
(b) Generate the embedded convolution matrix, by changing single dimensional data $y_N = (y_0, y_1, y_2 \ldots y_{N-1})$ into multidimensional series $(Y_0, Y_1, Y_2 \ldots Y_{N+M-1})^T = (y_{ij})_{i,j=0}^{M-1,N+M-1}$ with elements $(y_i, y_{i+1} \ldots\ldots, y_{i+M-1})$.
(c) Selection of any one of the following statistical parameter type:
switch type.
1. **Autocorrelation method of Length [$0 - N - 1 + M$] (Toeplitz structure)**
case {a, 'autocorrelation'}

$$Y_{auto} = Y((N+M-1):(M-1);$$

2. **Covariance method of Co-Variance; length [$M - N - 1$] (Hankel structure)**
case {b, 'cov-variance'}

$$Y_{cov} = Y(M+1:N,:);$$

3. **Forward-backward method of [(2(N + M)), M + 1)] (Toeplitz over Hankel structure)**
case {c, 'forward backward'}

$$\left[Y_{\text{fb}} = \frac{Y_{\text{cov}}}{\text{fold}\left(\left(Y_{\text{cov}}\right)^*\right)}\right]$$

end of switch statement.
Step [2]: Rank reduction using economy SVD
for $i = 1{:}K$
(a) Determine singular value decomposition (SVD) of Y having decomposition of the unitary matrix of the left singular value of size 'U' ($N \times (M+1)$), rectangular diagonal matrix 'S' ($(M+1) \times (M+1)$) and unitary matrix of right singular 'V' ($(M+1) \times (M+1)$).
(b) Set the diagonal elements of 'S' beyond the rank value, that is from $(r+1)$ to end equals to zero. Choose dominant diagonals in ascending order $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_2 \ldots\ldots \geqslant \sigma_r >> \sigma_{r+1} \ldots >> \sigma_d$, set $S(r+1\text{: end}, r+1\text{: end}) = 0$.
(c) Compute reduced rank SVD $Y_s = U \times Sr \times V'$.
(d) Construct first column of Y_s.
for column = 0: $p - 1$,
c = [c; mean (diag (Y_s, −k))];
end for column.
(e) Construct first row of Y_n
for row = 0: $q - 1$,
r = [r, mean (diag (Y_s, k))];
end for row.
Step [3]: Rank restoration of the structure of embedded matrix
switch type
case {a. 'Toeplitz'}
(a) **Toeplitz structure**
Compute row terms by calculating the mean value of the elements above the diagonal and compute column terms by finding the mean value of the elements below the diagonal.
Form a Toeplitz matrix with its column terms as its first column and row term as its first row of the matrix.

$$Y_s' = \begin{bmatrix} \frac{y_{11}+y_{22}+y_{33}}{3} & \frac{y_{12}+y_{23}}{2} & y_{13} \\ \frac{y_{21}+y_{32}}{2} & \frac{y_{11}+y_{22}+y_{33}}{3} & \frac{y_{12}+y_{23}}{2} \\ y_{31} & \frac{y_{21}+y_{32}}{2} & \frac{y_{11}+y_{22}+y_{33}}{3} \end{bmatrix}$$

case {b, 'Hankel'}
(b) Hankel structure
Let $Y_s = \text{flip}(Y_s)$ and $Y_s' = \text{flip}(Y_s')$ i.e. folded version of the row terms.
Compute the mean in the anti-diagonal form that forms a Hankel matrix (H).

$$Y_s' = \begin{bmatrix} y_{13} & \frac{y_{12}+y_{23}}{2} & \frac{y_{11}+y_{22}+y_{33}}{3} \\ \frac{y_{12}+y_{23}}{2} & \frac{y_{11}+y_{22}+y_{33}}{3} & \frac{y_{21}+y_{32}}{2} \\ \frac{y_{11}+y_{22}+y_{33}}{3} & \frac{y_{21}+y_{32}}{2} & y_{31} \end{bmatrix}$$

case {c, 'Topelitz over Hankel'}
(c) Topelitz over Hankel structure

(Continued.)

Algorithm 2. (Continued.)

(a) Let split the matrix Y_s as upper Hankel and lower Toeplitz one, $Y_s = \begin{bmatrix} \frac{X_u}{X_l} \end{bmatrix}$.
(b) Find the Hankel value of upper matrix $T_1 = \mathrm{Hankel}\,(X_u)$.
(c) Find Toeplitz value of the lower matrix $T_2 = \mathrm{Toeplitz}\,(X_l)$.
(d) Compute $T = \frac{T_1+T_2}{2}$ and $H = (\mathrm{flip}\,(T))^*$.
(e) Form a Toeplitz over Hankel matrix by concatenation of H with T.
(f) Find $Y_s' = [TH] = [H;T]$
end of switch statement.
Check whether the Frobenius norm is minimum or not
If $\| Y_s - Y_s' \|_F{}^2 = \min$
end if
end for loop of iteration k.
Step [4]: Single dimensional signal extraction
switch type
Type of embedded matrix
(a) **Extracting ACF:**
case {a, 'autocorrelation'}
$y = Y_s\,(1{:}\,p - q + 1,\, 1)$;
(b) **Extracting co-variance**
case {b, 'co-variance'}
$y = [\mathrm{flip}\,(Y_s\,(1,:).'); Y_s\,(2{:}\,\mathrm{end},\,1)]$
(c) **Extracting forward backward**
case {c, 'forward backward'}
$y = (Y_s\,(1{:}\,q/2,:),\,1)$
end of switch statement.

Output: The denoised path observables.

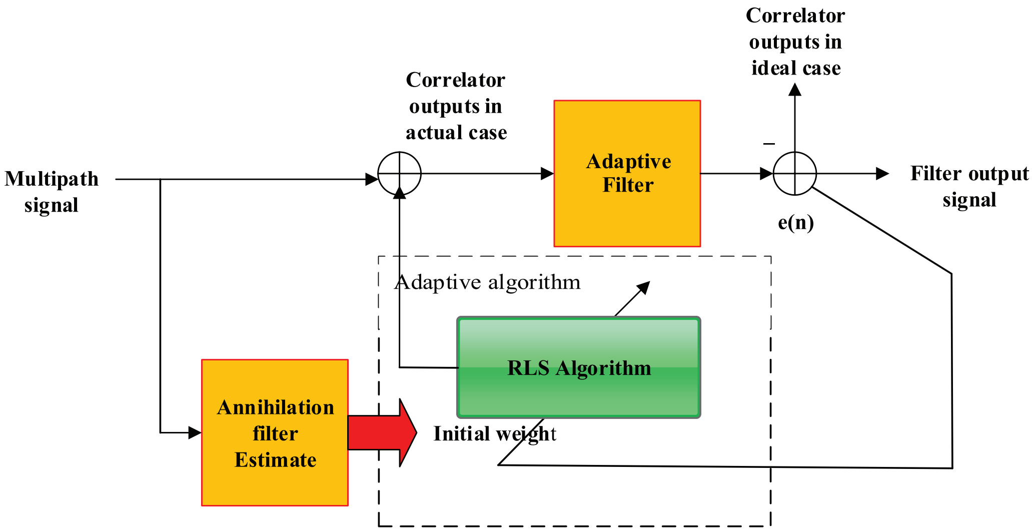

Figure 5. Estimation of multipath parameters via annihilation filter combined adaptive filter.

is subjected to a modified SOS-based multipath fading simulator. The software-based GPS receiver is used to examine the effect of the multipath signal in the code and carrier tracking loop. For evaluating the performance of the multipath estimation, ten multipath components are simulated, the doppler frequency, SNR, amplitude, and the phase of the path have been given as an input to the modified SOS multipath simulator. The simulation parameters of the annihilation filter model is illustrated in table 1.

Actually, the knowledge of a number of the multipath components is not known in advance to the receiver since it is a composite signal, but the actual number of estimated paths is set in the receiver. Therefore, in our simulation, we fix the estimated number of paths as a maximum when compared to the actual paths. There are several cases involved in analyzing the multipath components in the receiver side that are summarized below.

- Suppose for an instant the actual and estimated number of paths are the same then we declare as the success of mitigation on the receiver side.

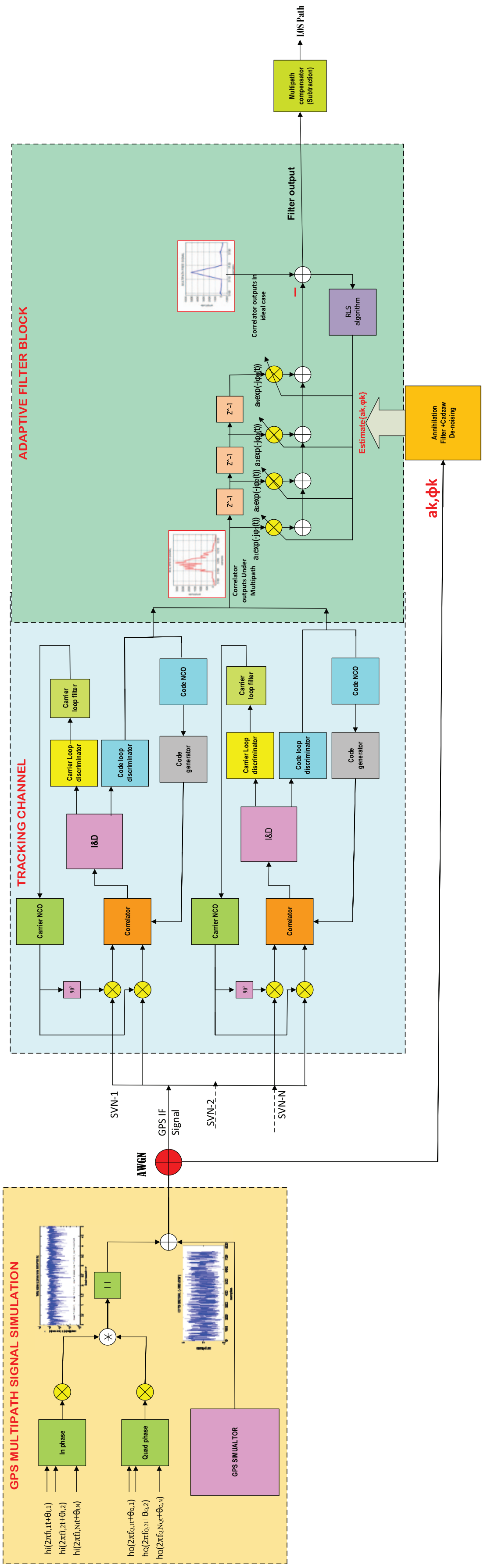

Figure 6. Incorporation of annihilation filter in GPS tracking channel.

Table 1. Simulation parameters used in the proposed multipath mitigation approach.

Parameters	Values
Number of paths (L_p)	10
Doppler frequency (f_d)	Two scenarios 100 Hz, 1000 Hz
Path phase φ (deg)	Random [0–2π]
Path normalized amplitude (power)	Decaying power delay profile co-efficient $\mu = 0.1$
Spacing of paths	Random varying from 0.01 to 0.35 chips (3.4230×10^{-7} s) Successive path spacing with a step of 0.0628 chips
Sampling factor (L)	5714 samples
Sampling rate (N)	51 samples
Period of $x(t)$	1 ms

- When the actual path (L_p) is 3 and also the estimated path is set more than 3 ($L_p' > 3$) then also it is a success.
- If the actual path $L_p = 4$, but the estimated path set in the receiver side is $L_p' = 3$, then the mean square error is more and the receiver fails to mitigate the multipath component. Similarly, setting $L_p' = 4$ and 5 also leads to success.
- In case of $L_p = 5$, $L_p' = 3$ and 4 termed as a failure and $L_p' = 5$, then it is success in mitigation of multipath component.
- If $L_p = 6$ and the estimated number paths are set as $L_p' = 3$, 4, 5 then it is a failure in mitigation of the multipath component.

In this way, we can easily classify the difference between the true and the estimated number of paths on the receiver side by measuring the mean square error. Table 2 summarizes the MSE values of true versus estimated path scenarios. In figure 7, the x-axis represents number of estimated paths ($L_p' = 1$–10), and the mean squared error for three true paths ($L_p = 3$, 4 and 5) is plotted. From this result, it has been observed that the fixing of a greater number of estimated paths against true path in the receiver minimizing the mean squared error while considering a lesser number of estimated paths than the true path, then the error is more which in turn a failing in estimating the multipath component.

The estimated values of the path observables are depicted in figures 8(a) and (b). We can observe that the true value of the amplitude of the paths is nearly coinciding with the estimated path and almost the mean square error is negligible up to the 9th path. An abrupt increase in MSE found in the 10th path as shown in figure 9(b). But on the other hand, the phase has a little deviation with the number of paths, and the MSE is gradually raised as the number of paths increased from 5 to 10 in steps of one path which is illustrated in figure 9(a).

The performance of the annihilation filter is analyzed through Montecarlo simulations and the comparisons were made with and without cadzow filter by considering the SNR level from 10 to 40 dB. By choosing LP = 10 and N = 51, in this simulation, the MSE is computed for the aforementioned two scenarios. From figure 10, while at large SNR, the performance degradation becomes larger when annihilation filter alone used, but employing with cadzow filter, it improves the MSE performance up to 0.02 in between SNR of 10–30 dB.

The estimated equalizer is then convolved with the received signal to yield an estimation of the transmitted signal. The filter coefficient of the adaptive filter is shown in figure 11. The correlation peak of multipath-free and the multipath affected signal is shown in figures 12(a) and (b). The simulation results of the adaptive filter/inverse filter indicate that the correlation output does not have a perfect triangular-shaped one, it is just smoothing the multipath output of the correlation peak. But on the other hand, the annihilation filter estimated multipath observables given as an input to the adaptive filter exactly retained the shape of the correlation peak.

After estimating the multipath component, the discriminator output of each multipath signal is calculated. Then the resultant parts caused by multipath signals are subtracted from the total discriminator output of the received signal that is the output component due to multipath signal. In the separation of LOS signal (the dominant peak amplitude) and multipath signals, the first peak of channel estimation is termed as LOS signal, and the remaining are multipath signals.

In a dynamic multipath scenario, the code and carrier tracking loop error are calculated from the acquired SVN 17, the multipath components are added after 300 s of the simulation.

In the tracking phase of the software receiver, once the carrier has been wiped off, the input signal correlates with three code replicas. The output of correlators shown in figure 13. The in-phase and quadrature-phase output of early, late, and prompt correlators before applying to the annihilation filter show the frequency of the carrier replica drifts is quite more and it has the repeated maximum and minimum values. But the correlation results after applying to the annihilation filter hold the highest correlation energy and the correlation output is constant over time. At the start of the figure, it can be seen that the signal is in phase, this is because the acquisition algorithm acquires a good estimate of the incoming frequency. Then after 18–22 ms, the energy in the in-phase correlators starts to decrease. This is a result of the Doppler rate in the signal and decrement in the incoming signal's carrier frequency. It can also be seen that after 22 ms from the start of the tracking, the in-phase and quadrature correlators have almost the same energy. This means that the phase error with respect to the carrier frequency of the incoming signal and the replica frequency is $\pm 45°$. After 20 ms it can be seen that all the energy is in the quadrature correlators (phase shift of $\pm 90°$) and at 50 ms the phase of the carrier replica is off by $\pm 180°$.

The code tracking i.e. delay-locked loop (DLL) discriminator output of the filter is plotted in figures 14(a) and (b). Initially, without employing annihilation filter the code tracking error is above 0.5 chips after introducing annihilation filter in the tracking loop of the receiver, the tracking error is significantly reduced within 0.1 chips.

Similarly, the carrier phase error was also observed for both the cases when the carrier tracking loop alone is used the effect of multipath is more and the frequency error is oscillating

Table 2. MSE of true versus estimated paths.

	No. of true path = 3									
Estimated paths	1	2	3	4	5	6	7	8	9	10
(MSE) Annihilation filter	0.0583	0.0150	2.2087×10^{-30}	1.7437×10^{-29}	1.3289×10^{-28}	1.2936×10^{-29}	2.9167×10^{-29}	1.8274×10^{-29}	5.3119×10^{-30}	5.2950×10^{-30}
(MSE) LS	0.07734	0.0989	4.5656×10^{-3}	7.96546×10^{-13}	5.87657×10^{-16}	8.56435×10^{-21}	5.44678×10^{-21}	8.56443×10^{-21}	4.345678×10^{-21}	6.65656×10^{-23}
	No. of true path = 4									
Estimated paths	1	2	3	4	5	6	7	8	9	10
(MSE) Annihilation filter	0.1906	0.0628	0.0178	6.5280×10^{-29}	4.4679×10^{-29}	7.1056×10^{-29}	6.8848×10^{-30}	4.5516×10^{-29}	2.7825×10^{-29}	7.4394×10^{-29}
(MSE) LS	0.60765	0.1367	0.09877	0.45785	5.45467×10^{-16}	6.84356×10^{-19}	7.54357×10^{-19}	2.32466×10^{-19}	6.76654×10^{-19}	3.64564×10^{-19}
	No. of true path = 5									
Estimated paths	1	2	3	4	5	6	7	8	9	10
(MSE) Annihilation filter	0.3956	0.1972	0.1112	0.0347	9.3798×10^{-29}	3.8310×10^{-29}	6.8957×10^{-29}	5.6908×10^{-28}	2.1393×10^{-29}	1.2649×10^{-28}
(MSE) LS	0.95754	0.74867	0.3464	0.3644	6.97607×10^{-17}	3.89705×10^{-17}	6.5654×10^{-17}	9.54334×10^{-17}	7.65433×10^{-19}	3.65446×10^{-19}

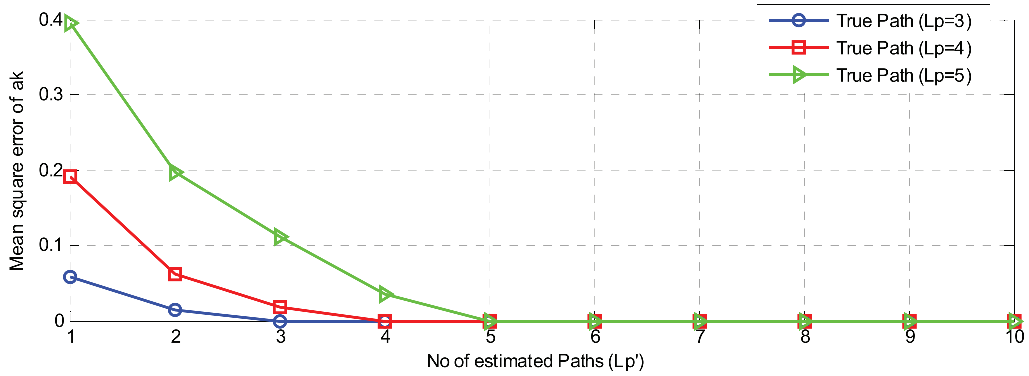

Figure 7. Mean square error of *ak* for true and estimated path.

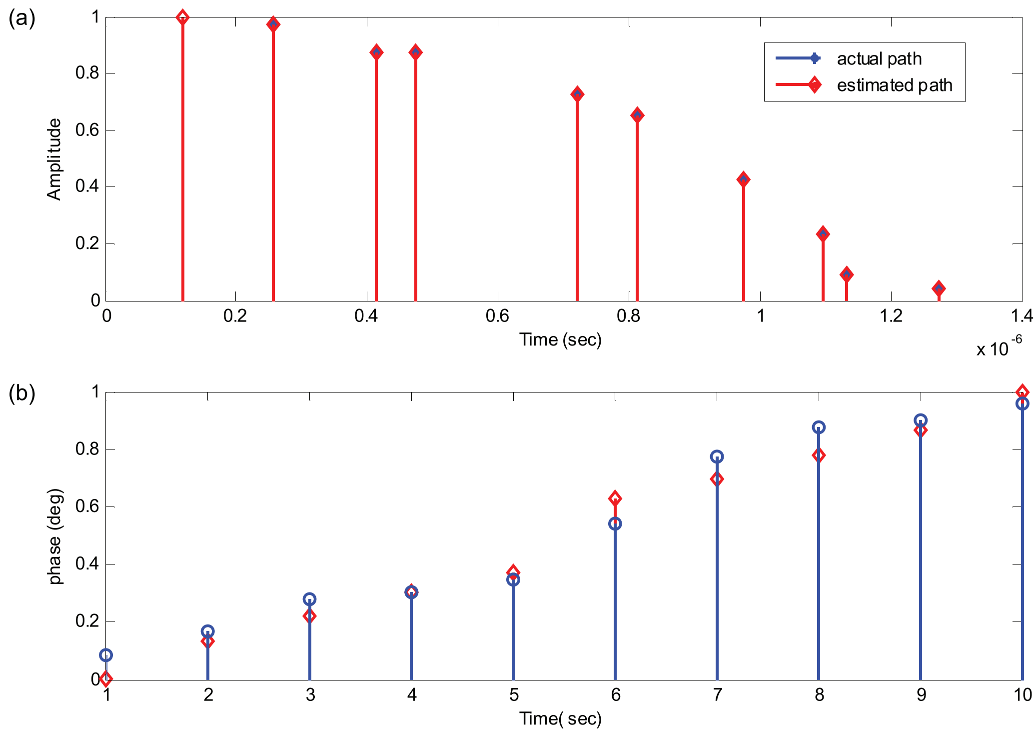

Figure 8. Estimation performance of amplitude and phase parameters using annihilation filter. (a) Amplitude. (b) Phase.

throughout the tracking time in the simulation. Since both loops are intercoupled, the lock has not been achieved in both the loops therefore one cannot demodulate the navigation data from the output of the tracking loop of the GPS receiver.

The above figures 15(a) and (b) depicting the frequency-lock loop discriminator output of the carrier tracking loop. One can see the frequency error observed nearly 100 ms approaches to zero, hence the lock has been obtained and the samples in the In-phase arm contain the navigation data information. This can be seen in figure 16. Thus, the usage of the annihilation filter effectively mitigates the multipath effect in ten path scenarios with different SNR levels when compared to the blind channel estimation algorithms.

The positioning error is also computed for both the scenarios. The positioning accuracy is involved with many parameters like number of visible satellites and its location geometry, atmospheric conditions, receiver clock bias, multipath etc. Here in this simulation of multipath components in GPS signal, totally six SVN's are simulated and a maximum of ten number of multipath components are added to the simulation for two SVN's (ten paths for SVN 17 and eight paths for SVN 23) and the remaining four SVN's do not have multipath effect. In the tracking module, each SVN is assigned to the tracking channel and their code phase and carrier error are computed without annihilation filter and the SVN 17 and 23 fail to achieve the lock and unable to demodulate the navigation data due to the multipath however, with only four satellites pseudorange measurements, the software receiver navigation module still could be able to compute the position error as shown in figure 17(a). On the other hand, after inserting annihilation filter to the tracking loop, it successfully

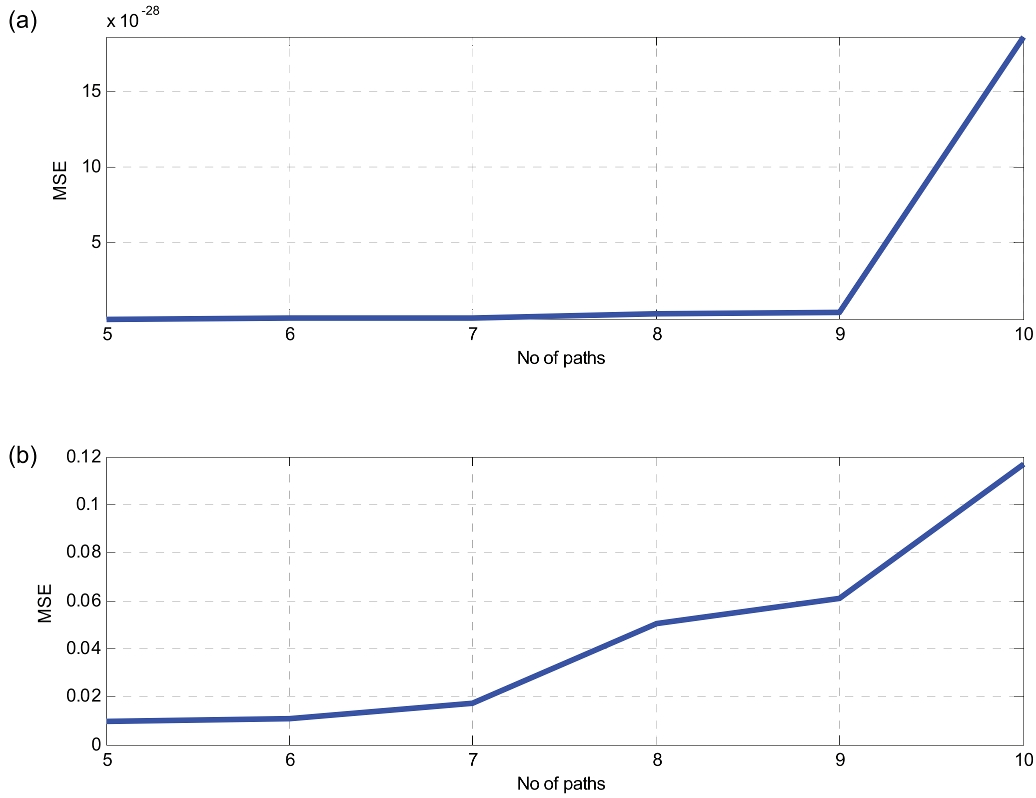

Figure 9. (a) Number of paths versus MSE (amplitude). (b) Number of paths versus MSE (phase).

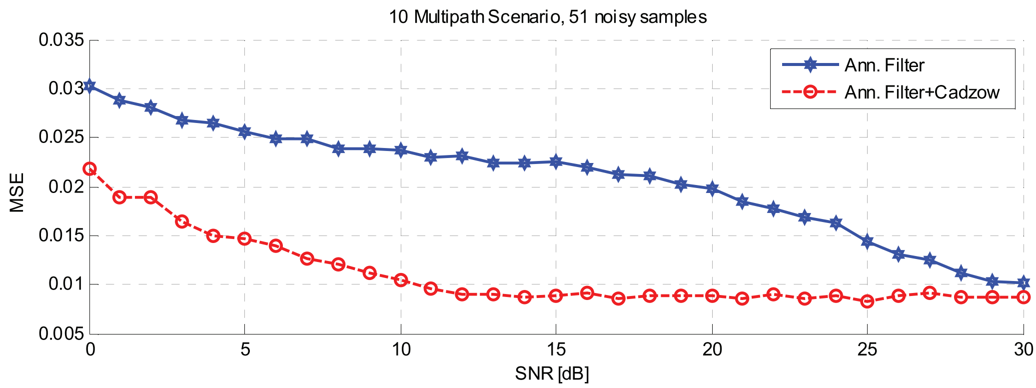

Figure 10. Comparison of SNR versus MSE with LP = 10 and N = 51, with and without cadzow algorithm.

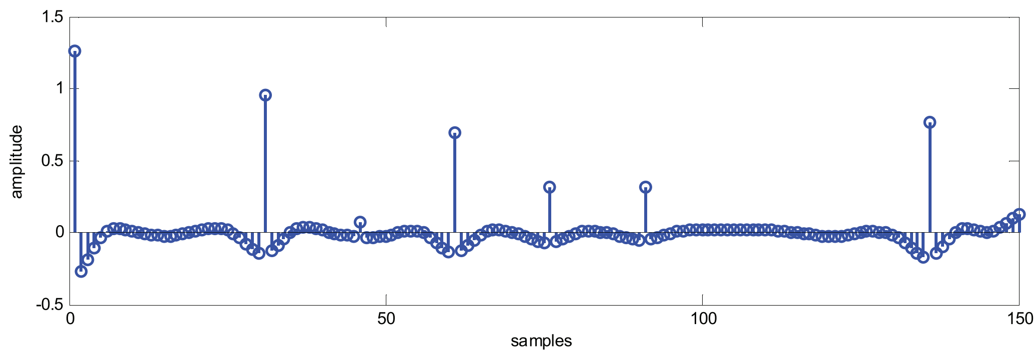

Figure 11. Impulse response of an inverse filter/adaptive filter.

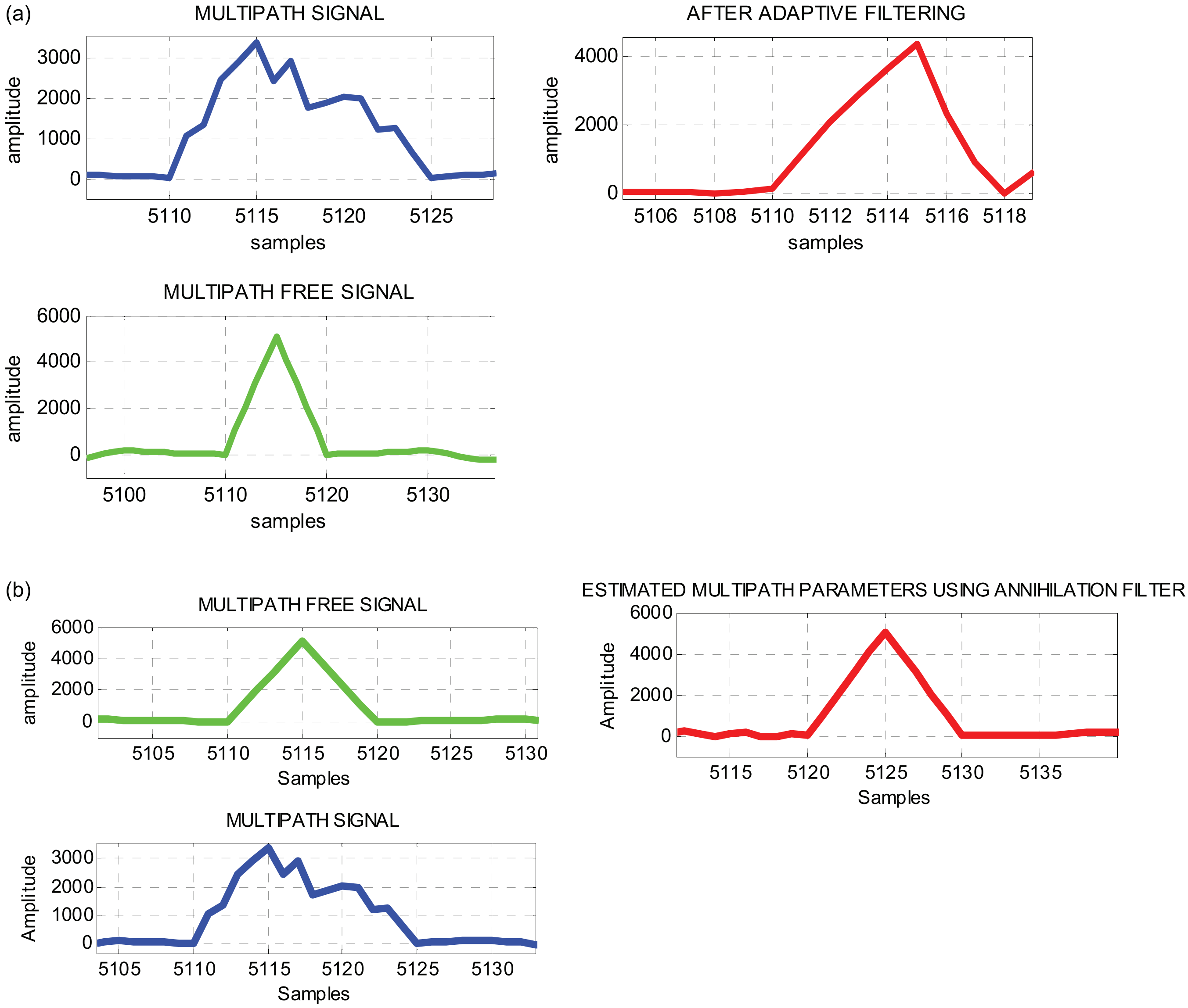

Figure 12. (a) Correlation results of least-square based estimation used in adaptive filters. (b) Correlation results of annihilation filter-based parameter estimation used in adaptive filters.

achieves the lock, and it has a better loop lock capability and can provide accurate loop parameters to decode the navigation solution for those multipath added SVN's (17 and 23) and compute the user position error with a reduction of mean value 9.3023 m as compared to without using annihilation filter case illustrated in figure 17(b).

In the future prospect of real-time implementation of the annihilation filter inserted in the tracking loop based on a high-performance DSP. The proposed multipath mitigation technique can be implemented in some of the recently developed economical DSP target boards like Tensilica Fusion F1 DSP from Cadence indeed may be the best choice. The aforementioned processor supports 110 MHz for full 12 channel satellite functionality that run independently and the required processor speed at the same frequency can be achieved using customized instructions of the processor more than three times low processing power compared to other DSPs. Most of the commercial hardware GNSS correlator chipsets require the presence of specialized cancellation hardware as it increases the hardware burden for the receiver design. Keeping this in mind, we first designed and tested the algorithms in MATLAB. In the next level, to confirm the correctness of the annihilation filter-based multipath mitigation technique on Tensilica Fusion F1 DSP complier, these algorithms will be implemented in 'C' language using recorded data. As the next process, the code will be compiled and loaded onto the DSP without any optimization further the optimization tools with different levels corresponding to algorithm, complier, and code level are carried out to ensure the lock-in state and the accuracy of tracking results. Finally, the optimized code will run on real-time live data that will effectively demodulate the navigation data.

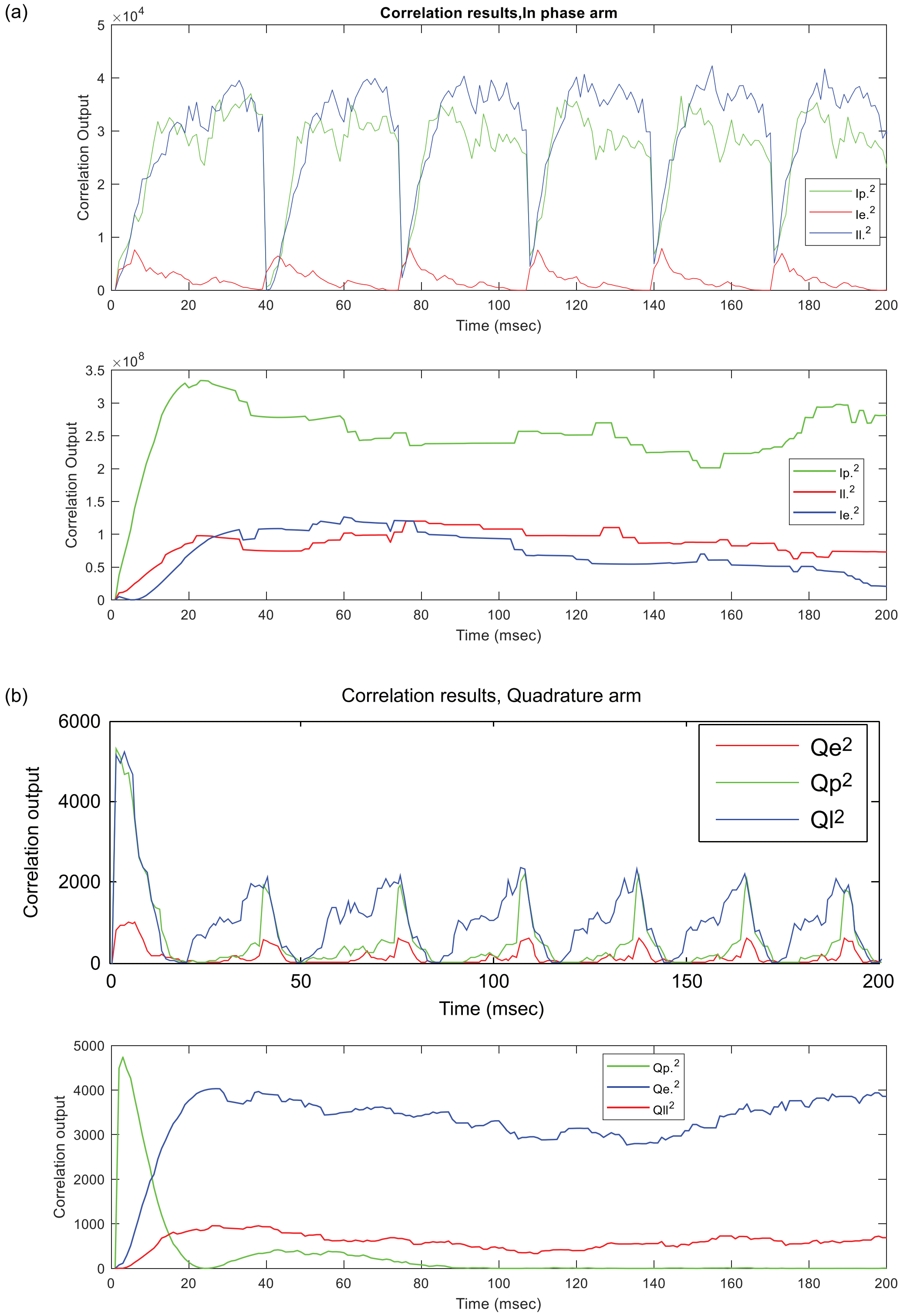

Figure 13. (a) Correlation results of in-phase arm before and after applying to the annihilation filter. (b) Correlation results of the quadrature-phase arm before applying to the annihilation filter.

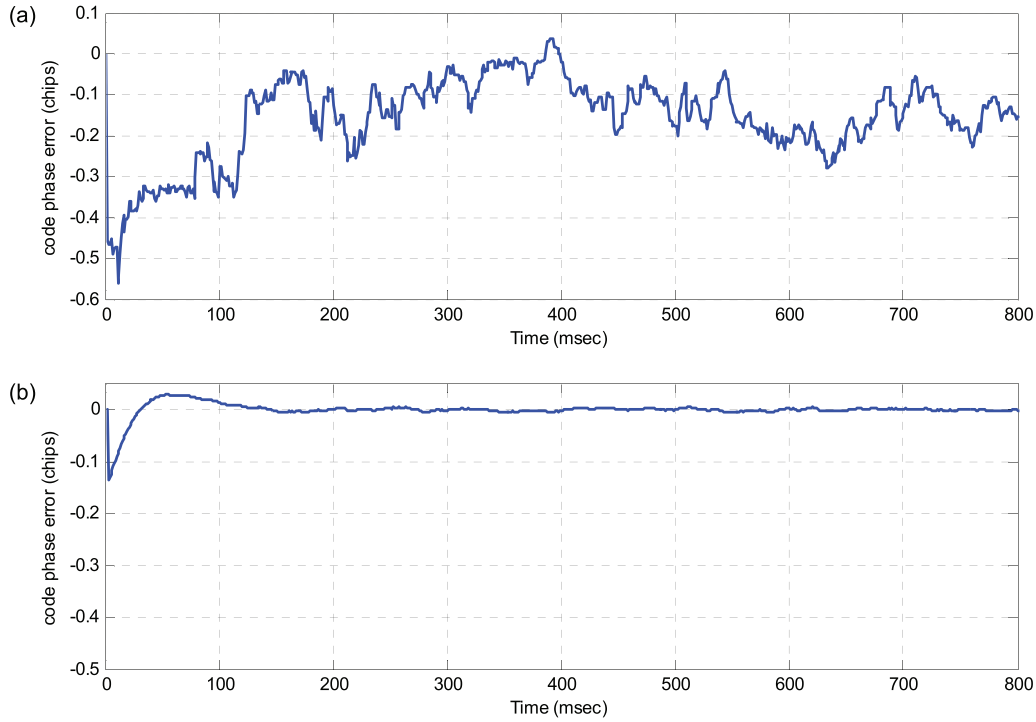

Figure 14. (a) Before multipath mitigation. (b) After multipath mitigation.

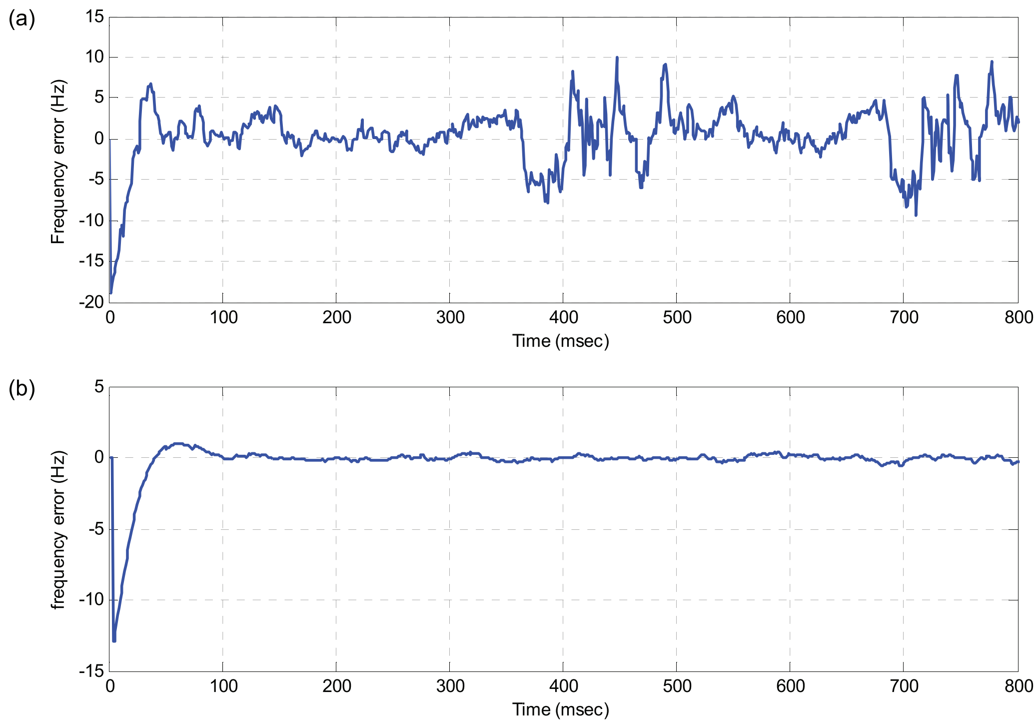

Figure 15. (a) Before multipath mitigation. (b) After multipath mitigation.

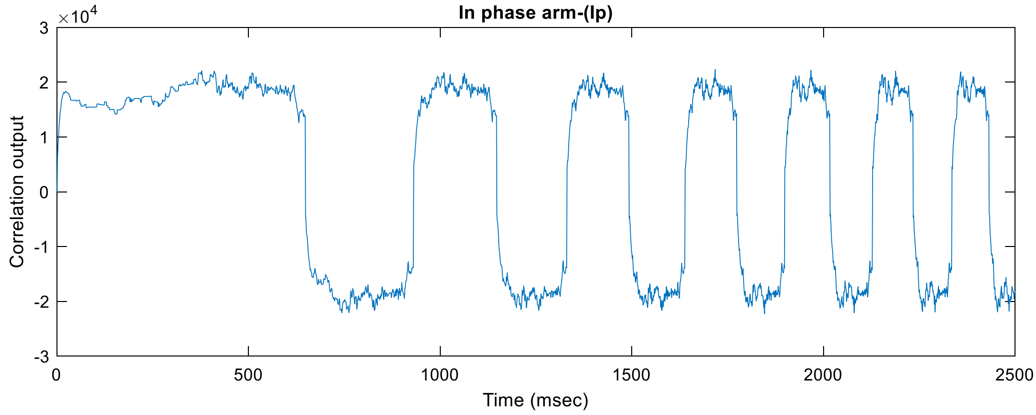

Figure 16. Tracking output values of the in-phase component (I_p).

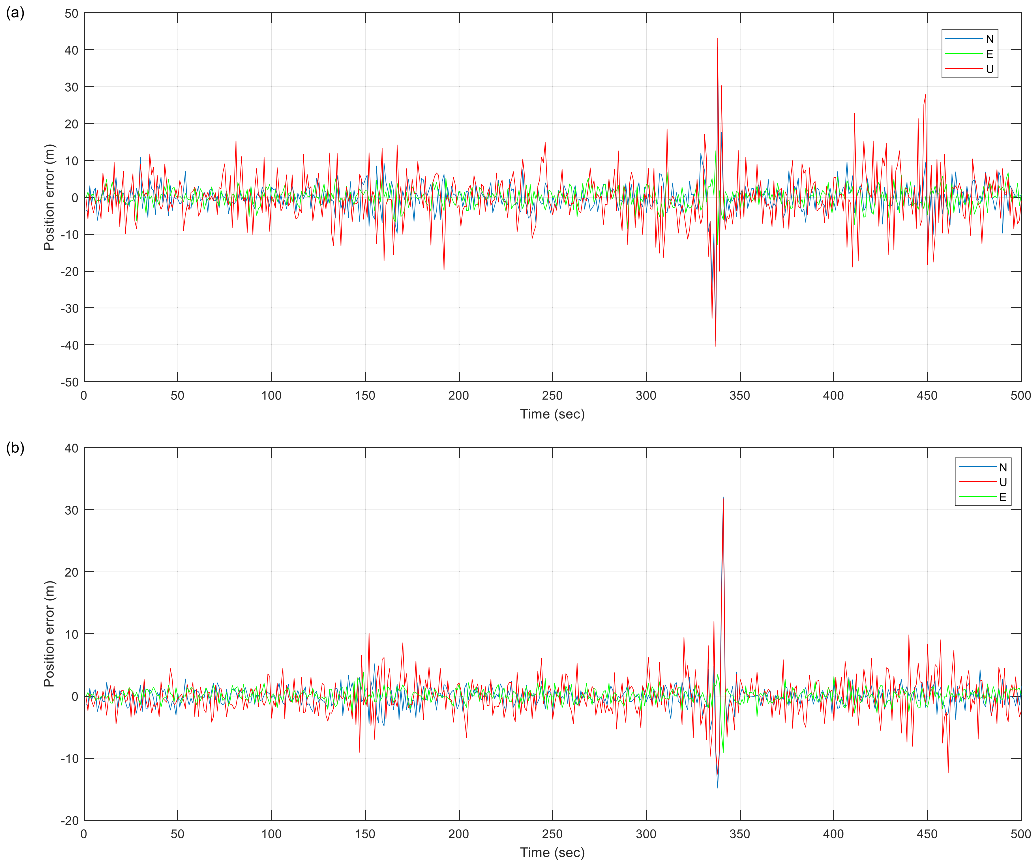

Figure 17. Positioning error computation. (a) Without annihilation filter. (b) After inserting annihilation filter in the tracking loop.

6. Conclusions

In this work, the annihilation filter is used for estimating the channel parameters by a smaller number of samples than a conventional Nyquist sampling. The total number of samples (N) in the receiver is chosen just more than twice the number of paths (L_p) to be estimated, that is 51 samples are considered instead of 5714 samples for estimating the ten number of paths. The estimation of path observables is found with a minimum mean square error of 0.02 when compared to the least squares and blind channel adaptive techniques. One interesting phenomenon can also be observed from the setting of the number of estimated paths in the receiver always chosen greater than the actual number of paths that minimizes the MSE. Fixing the lesser number of estimated paths against the true path leads to a higher mean square error, from this information, the adjustment of adaptive path setting is being done on the receiver side easily to estimate the channel.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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Conflict of interest

The authors declare no conflict of interest.

Author contributions

The two authors equally contributed towards the estimation of Multipath channel parameters using annihilation filters in software GPS Receiver's Tracking Module. Moreover, Arul Elango has formulated the theoretical developments and the simulation analysis along with Professor Landry. A E drafted and R J reviewed the paper.

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